**Calculus in Context I**
**Developed by Sarah Hews**

The following describes my innovative Calculus in Context I class. For further information please contact me at shews@hampshire.edu.

*Narrative (Phase 1)*
The narrative is built through the first two classes where students learn about the importance of understanding how things change and why we model through an interactive game of a disease outbreak. We build a simple differential equation (rate of change) model of the disease outbreak and discuss why we would want to *solve* them. I lay out that there are two main concepts that are important to understand so that we can *solve* these rate of change equations: limits/successive approximations and instantaneous rate of change/derivative. The class is set up to thoroughly learn these two concepts so that we can solve differential equation models. Toward the end of the semester, students will use these two concepts to solve one of a selection of case studies that include a mathematical model.

*Concepts/Content (Phase 2)*
Students build their understanding and intuition of limits through successive approximations. We briefly discuss vertical asymptotes and holes but only from a graphical perspective. Once students have grasped the concepts of limits through successive approximations, we discuss the importance of rates of change. Starting with average rates of change, students learn to take successive approximations to find an instantaneous rate of change. The definition of a derivative (instantaneous rate of change) is therefore intuitive and a natural extension of limits. We take a brief detour to listen to a podcast on Newton and Leibniz appreciating the history behind the discover of Calculus. From there we explore local linearity and what it means to be able to take a derivative. We naturally move to the connection between the value of the derivative and the slope or steepness of the curve using hiking analogies. We conclude the big picture concepts by concluding that it would be nice to have equations to find the derivative/instantaneous rate of change at any point.

*Skills (Phase 3)*
Once students understand what a limit is and how to use successive approximations to find it and what a derivative is and how to use successive approximations to find it we dig into using some algebra to make some of our calculations simpler. Motivation for finding some limits algebraically is established and covered. We derive the differentiation rules and how to use built in commands on various software to find derivatives.

*Concepts/Content (Phase 4)*
Once students are comfortable with calculating derivatives we revisit Concepts/Content (Phase 2) to make sure we haven't lost what a limit is and what a derivative is. Once we review these ideas we progress to graphing functions and their derivatives, being comfortable moving between them and sketching them. We revisit the relationship between the derivative and the steepness of the curve and then explore what the derivative of the derivative is and why it is useful to understand the function. We naturally articulate the

first and second derivative rules and move quickly to using that information to understand functions in general (sketching them using first and second derivative information). These are all presented intuitively and from a conceptual understanding rather than skills. Local maximums and minimums naturally emerge as we explore different functions and the derivatives of different functions. Students discover and articulate the relationship between first and second derivatives and local mins/maxs. From there it's a natural next step to briefly explore optimization problems.

*Applications/Optimization (Phase 5)*
We do spend some time on optimization problems and the power of using derivatives to optimize functions. We dig into real world applications and again keep in mind the concepts rather than focusing on algorithms.

*Models/Narrative (Phase 6)*
Now that the two main concepts are established (limits and derivatives) and the power of successive approximations, as a class we explore a well established coral reef model. Students learn how to read the differential (rate of change) equations using my 'model breakdown' method. We build the intuition and story of the model taking care to graph each rate and understand the relationships between coral reef and macro algae. To solve the model, we exploit the concept of local linearity to discover Euler's Method. We take a brief detour to listen to a podcast on the impact of Euler in mathematics more generally. Now that we have articulated Euler's Method, we discuss how successive approximations of Euler's method leads us to solutions (or what we believe are solutions). When the tediousness of doing this is established we appreciate the power of computer programs. Learning some basics of MATLAB we learn how to solve systems of differential equations using Euler's Method. The limitations of Euler's Method are briefly mentioned but primarily the power of our concepts of Calculus to solve this systems is emphasized.

*Case Studies/Narrative (Phase 7)*
To practice applying understanding of these concepts, students after selecting preferences are placed in a group to complete a case study. Case studies range from adapting Dengue Models for the Zika Virus to adapting an HIV model to explore CRISPR, etc. Each case study requires that students read and explain the model using the 'model breakdown' method, run simulations using MATLAB and Euler's Method, modify the model slightly demonstrating a strong understanding of rates of change, and communicate this work (in writing and through presentations). Students working in groups allows a collaborative atmosphere that is supportive and encouraging. Each case study is designed so that students can continue the work past the end of the semester and perhaps even publish the work.

*Conclusion/Narrative (Phase 8)*
I take care at the end of the semester to have the students lay out all of the concepts that they have learned and how they fit in the larger picture. Students have to articulate the difference between concepts and skills and demonstrate that they understand why these concepts are relevant for different fields and their everyday lives. Successive approximations is emphasized as a tool that will continue through Calculus in Context II if students chose to continue on.